Lenses: Optical Basics

Figure L-1 shows the principle of a lens. Rays from an infinite light source appear to be parallel, as illustrated in figure L-1a. A lens which is positioned perpendicularly to these parallel rays (i.e. the rays and the optical axis are parallel) brings them together at a point, known as the focal point. In other words, one could say that the focal point is a mapping of an infinitely remote light source. The distance between the lens and focal point is the focal length f. Thus if we want to capture an infinitely remote object on a CCD chip, the distance between the lens and the chip has to be exactly that of the focal length. In other words, the CCD chip has to lie in the focal plane. Imagine that our light source is moved closer to the lens (b). Now the lens brings the rays together behind the focal point and therefore the distance between the lens and the chip has to be increased in order to get a sharp image. These considerations apply not just to ideal thin lenses, but also to actual compound-lens systems (see following paragraphs). When using lenses for high precision applications, it is important to differentiate between the ideal thin lens principles and those of real compound lens systems. For everyday applications, the principles of the ideal thin lens can be applied to normal compound lens systems.

Hence focussing a lens means changing the distance between the lens itself and the CCD chip. The variation of this distance is subject to mechanical constraints. Usually a lens is capable of focussing from infinity to the so-called minimum object distance (MOD). The MOD can be decreased by installing an extension tube (sometimes called macro ring) between the lens and the camera. With this extension tube in place it is not possible to focus on infinitely remote objects. The longer the extension tube, the smaller the minimum and the maximum object distances become. Unfortunately the maximum object distance decreases faster than the minimum object distance which has the result that focussing on certain points becomes impossible. Figure L-7 exemplifies these parameters for the case of C-mount lenses (see below).

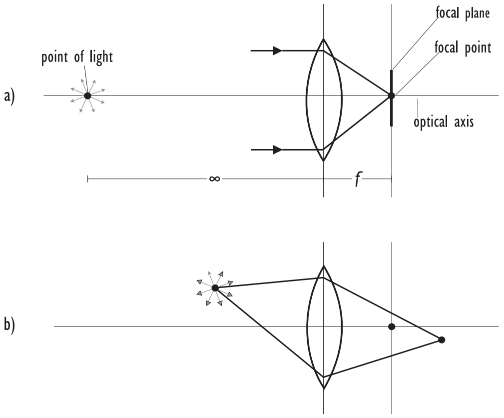

Figure L-1 shows the principle of a lens. Rays generated by a light point are parallel in infinite distance from that point (a). A lens which is positioned perpendicularly to these parallel rays bundles them in the focal point. Thus the focal point is a mapping of the infinitely remote light point. The distance between lens and focal point is the focal length f. lens (b). Now the lens bundles the rays behind the focal point and therefore the distance between lens and chip has to be increased to get a sharp image

When it come to zoom lenses, the usage of extension tubes is not a good idea. Here close-up lenses or macro lenses can be of assistance (see below for further details).

Figure L-2 shows the basic lens formulae as defined by Descartes for the so-called thin lens (which regards a lens as infinitely thin). In everyday practice we often need to determine the focal length. From the basic laws:

$$\frac{1}{b}+\frac{1}{g}=\frac{1}{f}$$

$$\frac{B}{G}=\frac{b}{g}$$

we derive (using $b = gB/G$)

$$f=\frac{gB}{G+B}=\frac{g}{1+\dfrac{G}{B}} \qquad \text{(L-1)}$$

where b and g are the distances between the lens and the image and the lens and the object (see figure L-2) while B and G describe the size of the image and the object. The quotient from B and G is the so-called magnification factor m:

$$m=\frac{B}{G}$$

The shorter the focal length, the more the lens refracts the rays of light. The so-called power D of a lens is therefore reciprocal to the focal length thus:

$$D=\frac{1}{f}$$

The unit of refractive power is known as the diopter (dpt). The unit for the focal length is the meter (m). Thus a lens with a refracting power of 10dpt has a focal length of 100mm.

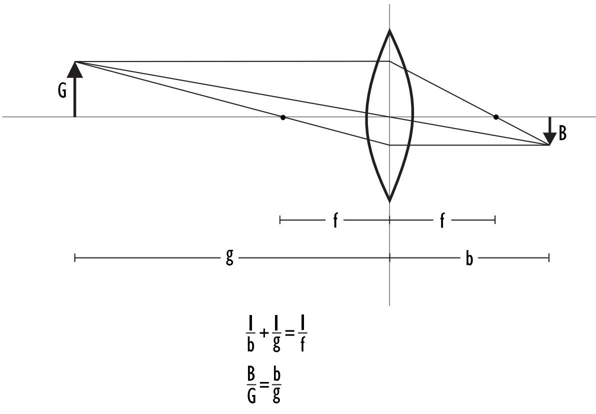

Figure L-2: The lens law

Besides the focal length, the angle of view is another important feature of a lens. The angle of view q is defined as (see figure L-3)

$$\theta = 2\tan^{-1}\frac{B_{\max}}{2f}$$

where $B_{\max}$ is one of the dimensions (horizontal, vertical or

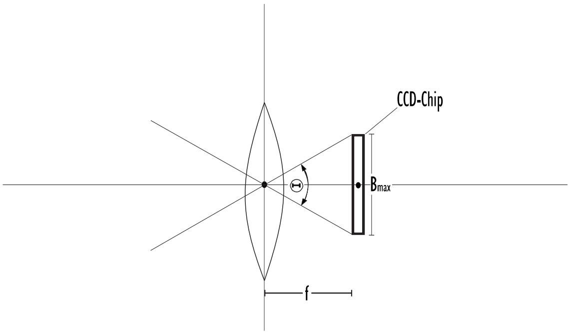

Figure L-3: The field of view

diagonal) of the CCD chip. A lens is sometimes classified according to its angle of view. In practice this classification is not very strict. Typical values of a diagonal angle of view are:

100°	super-wide angle
65°	wide angle
35°	standard wide angle
20°	standard
12°	light telephoto
6°	telephoto

As the distortion of a lens increases with its angle of view, these optical parameters not only describe a lens, but also act an indication of quality. For the purposes of metrology wide angle lenses should be of high quality while super-wide angle lenses should be avoided at all costs. In the case of C-mount lenses the 'dangerous' angle of view corresponds roughly to a focal length of 8mm. For metrology, lenses with a focal length of less than 8mm should never be used.

Another important parameter is the depth of field. Figure L-4a depicts the root of this problem. If a CCD chip is positioned to capture a sharp image from a point A, a closer point B results in the so-called blur spot. Its acceptable diameter (sometimes called circle of least confusion) is heavily dependant upon the application. In the case of sub-pixel arithmetic, unsharp (i.e. smooth) graylevel transitions are even desirable. Nevertheless, for visualization purposes we need images which are as sharp as possible. As a rule of thumb the diameter of the blur spot should not be larger than the diameter of the CCD pixel (a typical value is 10mm).

Figure L-4b shows why an aperture (also called iris) decreases the blur spot. An aperture is described by its F-stop k which is defined as:

$$k = \frac{f}{d}$$

where d is the effective diameter or the clear aperture of the lens. In the theoretical case of an aperture in front of a thin lens (see beginning of this section) the clear aperture is equal to the diameter of the aperture itself. In real lenses, however, this is a little more complicated, but hidden for the user. He or she only has to deal with the values:

k = 0.71, 1, 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22, 32

(each increase of F-stop halves the amount of light captured by the lens). Reflex cameras offer this so-called 'step-by-step' adjustment, but in the case of lenses designed for CCD cameras, the aperture is infinitely variable. A small aperture (corresponding to a high F-stop), a small focal length and a large object distance lead to a small blur spot. With u' as the diameter of the blur spot, the exact limits of depth of field are the magnitudes of the near (|g|v) and the far (|g|h) limit:

$$|g|_v = \frac{|g|}{1 + u'k\,\dfrac{|g| - f}{f^2}}$$

$$|g|_h = \frac{|g|}{1 - u'k\,\dfrac{|g| - f}{f^2}}$$

If we want to calculate the magnitude of g (the distance between the lens and the object) and the F-stop k we use

$$|g| = 2\frac{|g|_v |g|_h}{|g|_v + |g|_h} \qquad \text{(L-1)}$$

and

$$k = \frac{f^2}{u'}\,\frac{|g|_h - |g|_v}{2|g|_v|g|_h - f\left(|g|_v + |g|_h\right)} \qquad \text{(L-2)}$$

When talking about macro lenses the opening is described with the so-called numerical aperture (see below):

$$N.A. = n \sin\frac{\theta}{2}$$

Here q is the angle of view (as described above in figure L-3) and n is the calculation index of the material which surrounds the lens. For air n=1. The relationship between the relative opening k and the numerical aperture can be approximated with:

$$k \cong \frac{1}{2N.A.}$$

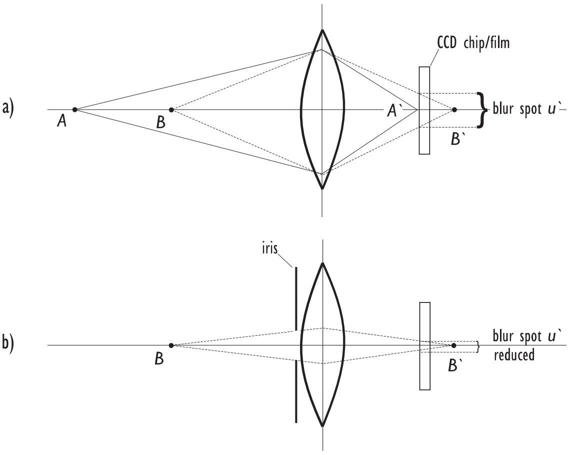

Figure L-4: illustrates a CCD chip where point A is in focus and thus the nearer point B is mapped to a blur spot (a). An apature can reduce this diameter.

C-mount lens

Beyond the optical basics discussed in the previous section, in practice we need to deal with a few additional parameters. First of all there are many different methods of mounting a lens on a camera. Fortunately computer vision is dominated by C- or CS-mount lenses and cameras (see figure L-5). There is only one important exception: line-scan cameras with 2048 or more pixels need a larger image format than C-mount lenses can offer. In this case SLR lenses can usually be used. For in depth information about line-scan cameras, take a look at the introductory notes to the camera section of this catalogue.

The only difference between C- and CS-mount lenses is their flange back distance or flange focus distance which is the distance between the end of the lens' screw thread and the focal plane (see figure L-5). The flange back distance is 17.5mm for C-mount and 12.5mm for CS-mount lenses. Thus, applying an extension tube of 5mm to a C-mount lens, changes it into a CS-mount lens. As the C-mount standard is internationally accepted, there is a wide range of lenses available. There is one important exception to this rule: C-mount 3CCD cameras (see camera section) do not work with any C-mount lens. From the mechanical point of view the flange back distance (see figure L-5) of various C-mount lenses is too long (never try to mount such a lens, it may damage the prism in the camera). Additionally the optical properties of C-mount lenses do not always fit the optical properties of the 3CCD camera. Therefore it is important to consider the compatibility lists offered by the camera manufacturers.

C-mount lenses have their origin in the times of pick-up tube cameras. These tubes were available with major diameters of 1/2", 2/3" and 1". The longer side of the tube's photosensitive area equals half the size of the tube diameter (see figure L-6). Modern CCD chips are becoming smaller and smaller. The usual sizes (also known as chip format or camera format) are 1/3" and 1/2". Different chip formats require corresponding lens formats. Generally the lens format has always to be equal or larger than the chip format. Especially in the context of metrology it is a good idea to use larger lens formats, since lens distortions are high near their rim.

The equations at the beginning of this section suggest that the optical parameters are freely selectable. From a technical point of view this is true. Nearly every lens can be custom designed. This is, however, far too expensive for everyday practice. Figure L-6 gives a short survey of C-mount lenses available on the market.

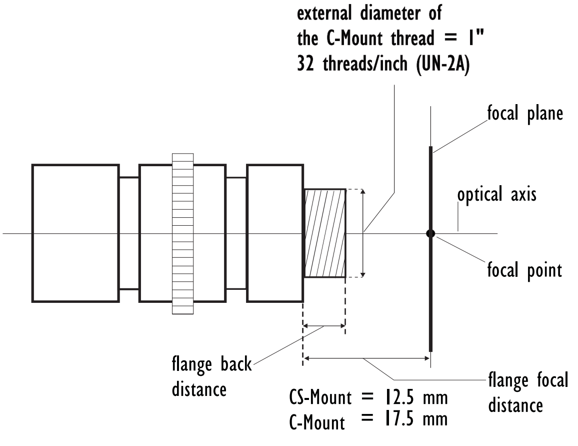

Figure L-5: The mechanical parameters of C and CS mount lenses.

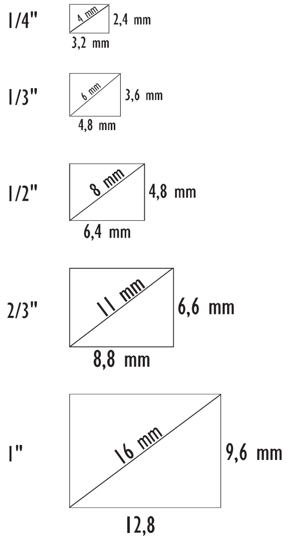

Figure 6: the C-mount standard offers varing sizes of light sensitive areas. The inch measurement corresponds to the external diameter of pick-up tube cameras. The larger side of the light sensitive area corresponds to approximatly half that of the diameter of the pick-up tubes

As previously discussed, every lens has a minimum object distance (or MOD). Figure L-7 depicts the connection between focal length, the normal MOD and the change of the MOD due to the application of an extension tube.

Close-up lenses

With the exception of inexpensive lenses, they have a screw thread for the attachment of filters. In the place of these filters, close-up lenses can be attached. They decrease the minimum object distance of lenses thereby having the same effect as extension tubes. Their utilization with zoom lenses, however, is not to be recommended or rather too much trouble. Figure L-8 depicts the principle of a close-up lens. A compound lens system without a close-up lens add-on (a) focuses on an infinitely-distanced light source at its focal point. If we attach a close-up lens with a power of 2 diotpres (it is usual to describe close-up lenses with their refracting power rather than with their focal length) to the front of this lens we have to move the light source to the focal point of the close-up lens (0.5 m in our example) in order to maintain the focus at the lens' focal point.

If the calibrated distance of the lens is shorter than infinity, it obviously shortens the object distance.

With the lens' calibrated distance g, the focal length of the close-up lens fN and the distance between the close-up lens and the lens itself d, the new object distance can be calculated:

$$g_N = f_N \frac{g-d}{f_N+g-d}$$

The new focal length fNO for the optical system constructed from the lens (focal length fO) and close-up lens (focal length fN) is:

	f-length	12mm	16mm	25mm	50mm	75mm
	MOD	20cm	30cm	50cm	100cm	100cm
macro ring	0.5mm	12..31cm	22.54cm	41..129cm		
	1.0mm	8..15cm	17..28cm	32..66cm		
	1.5mm	6..10cm	14..20cm	27..45cm	75..175cm	
	5.0mm	2..3cm	7..8cm	14..16cm	43..59cm	69..125cm
	10.0mm			9..10cm	29..34cm	50..69cm
	15.0mm				23..25cm	41..50cm
	20.0mm					35..41cm
	25.0mm					30..35cm

Figure L-7: Relationship between focal length, normal MOD and reduction when a macro ring is used.

$$\frac{1}{f_{NO}} = \frac{1}{f_N} + \frac{1}{f_O}$$

As we are faced with a whole multitude of different units, our life is made more simple in the following way (units of measurement are in []):

$$g_{N[\mathrm{cm}]} \cong \frac{100}{D_{N[\mathrm{dpt}]} + \frac{1}{g_{[\mathrm{m}]}}}$$

The distance between the close-up lens and the lens d is a matter of a few centimeters and can therefore be disregarded for day to day applications. The table as shown in figure L-8 was calculated with the help of this formula. Moreover the calculation of the new focal length, is obtained by the slight modification of the original formula:

$$f_{NO[\mathrm{mm}]} = \frac{1000}{D_{N[\mathrm{dpt}]} + \frac{1000}{f_{O[\mathrm{mm}]}}}$$

The refracting power of a close-up lens must never exceed 20% of the refracting power of the lens itself. If this is kept in mind, it is possible to keep distortion levels under control. If the required reduction of the object distance cannot be achieved in this way or a high precision metrology application is being setup, particular attention should be paid to the following section about macro lenses.

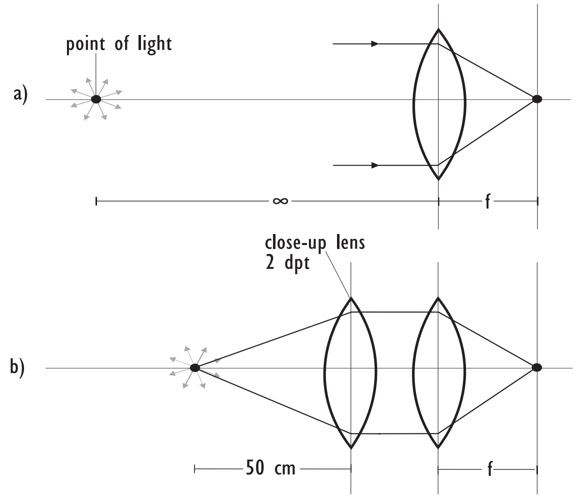

Fig L-8: The effect of a close-up lens: A lens without a close-up lens focuses to an infinitely distant focal point. When we screw on a close-up lens with the refracting power of 2 dpt (as shown in b), we have to move the focal point of the close-up lens, in order to keep the subject in focus.

Macro lenses

If the application of an extension tube or a close-up lens can not achieve the required results, a macro lens can be used instead. Macro lenses cover magnification factors of between 0.1 and 10. They are used almost exclusively in metrology and are therefore very precise and robust. They are fitted out with neither an adjustable aperture nor an adjustable focus - the object distance is fixed. As this is the case the lighting of the subject being photographed and its mechanical construction play an important role and must be totally befitting to the lens. In contrary to normal lenses the principal characteristic of a macro lens is not its focal length, but is magnification factor. This saves certain calculations. The opening of a macro lens is usually expressed through a numerical aperture setting (see previous sections). One can assume that this setting is about the reciprocal value of the doubled aperture number.

Telecentric Lenses

When it comes to measuring the depth of a certain object, we are confronted with the problem of perspective distortion. Take a look at figure L-10 for an example, which illustrates that this distortion cannot always be calculated out.

This is the role of the telecentric lens. The telecentric lens' aperture is positioned directly at its focal point (see figure L-11). Hence, only parallel (or almost parallel) rays are able to pass through this aperture and therefore as the reflecting object seems to be infinitely remote there can be no perspective distortion. This technique brings with it a certain disadvantage: The diameter of a telecentric lens has to be larger than the object being measured. In practice, however, true telecentry can never be obtained. Thus one of the most important parameters of a telecentric lens is its telecentric depth. Every telecentric lens has ist optimal object distance. Moving the object nearer of further away from the telecentric lens causes the mapping on the CCD chip to vary. This variation should not exceed 1mm. The telecentric depth is the area in which the object may be moved, so that it does not cause the mapping of the object on the CCD chip to vary more than 1mm. Some manufactures allow for a telecentric

Refracting power of close up lens	Calibrated lens distance			
	∞	3m	1m	0.5
1 dpt	100cm	75cm	50cm	33cm
2 dpt	50cm	43cm	33cm	25cm
3 dpt	33cm	30cm	25cm	20cm
5 dpt	20cm	19cm	16cm	14cm
10 dpt	10cm	10cm	9cm	8cm

Figure L-9: This table shows the relationship between the original calibration distance of the lens, close up lens refracting power and the resulting new object distance.

depth variation of 10mm.

As is also true for the macro lenses, as described in the previous sections, lighting and the setup of camera and object play a key role in the success of your application when you use a telecentric lens.

Examples

Example 1: The image of a 50mm * 50mm object must cover the CCD chip of an 1/2" camera as much as possible. The distance between the object and the lens is 150mm. What focal length does a 1/2" lens have to have?

Using equation L-1 where g = 150mm, B = 6.4mm (which is the height of an 1/2" CCD chip) and G = 50mm we get f = 13mm. Page XX shows the availability of a 2/3" lens where f=16mm. The rule has already been fulfilled where by the lens format has to be larger or at least the same size of the CCD chip.

Table L-7 shows the MOD for a 16mm lens at 300mm. Therefore a 1.5 mm macro ring has to be implemented to reduce the MOD to 140mm. The maximum object distance is 200mm.

The table to the right has been designed to help you find the correct lens for your application without the need for lengthy calculations.

Summary

- A small aperture opening, a short focal length and a large object distance lead to a deep field of depth.
- The diameter of the blur spot should never be larger than the edge of the CCD chip pixels (typical value 10mm)
- Lenses with a focal length of less that 8mm should not be used for metrology, because of unacceptable distortion levels.
- The lens format must be larger or equal to the format of the CCD chip.
- Distortion is particularly problematic towards the rim of the lens, which is why the lens format should always be larger than that of the CCD chip when being used for metrology applications.
- Extension tubes and close-up lenses reduce the minimal object distance for lenses. For zoom lenses the application of an extension tube should be avoided.

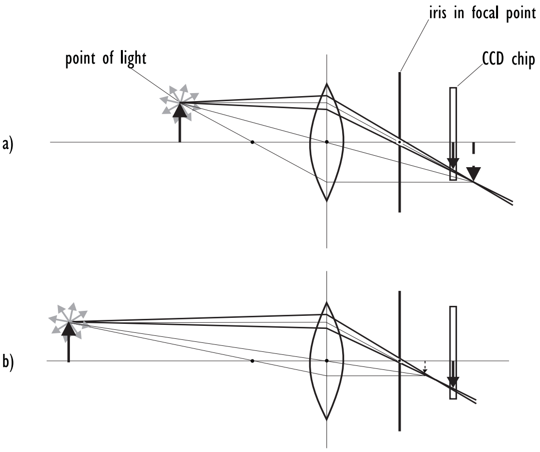

Figure L-11: When it come to measuring the depth of a certain object, we are confronted with the problem of perspective distortion. This problem can be combatted with the application of a telecentric lens. The telecentric lens' aperture is positioned directly at its focal point.

- The refracting power of a close-up lens must never exceed 20% of the refracting power of the lens itself. If this is kept in mind, it is possible to keep distortion levels under control. If the required reduction of the object distance cannot be achieved in this way or a high precision metrology application is being setup, a macro lens should be used.
- Telecentric lenses avoid the problem of perspective distortion. Their diameter, however, must be at least that of the object which is being measured.
- For industrial applications, you must not cut costs on mechanical stability. In this field, lenses which can have their focus, aperture setting and zoom etc. screwed into place once they have been set are particularly recommended.
- C-mount 3CCD chip cameras have different optical and mechanical features from normal one chip CCD cameras. For this reason, it is important to pay particular attention to the manufacture's list of compatible lenses.

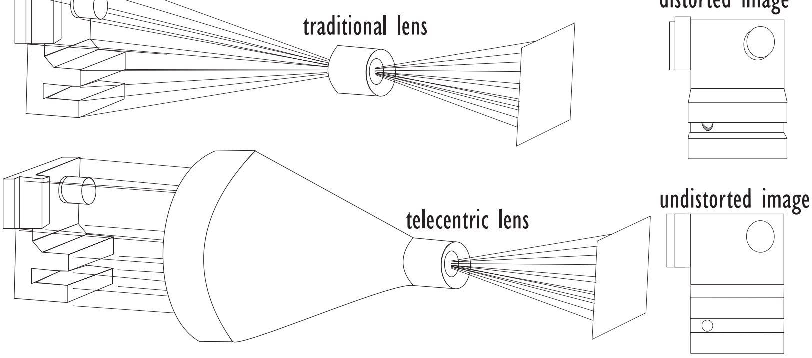

Figure L-10: The mapping of three dimentional objects onto a two dimentional leben with a telecentric lens is based on a parallel projection. The resulting image appears like an orthographic projection (diagram reporduced with kind permission from Rodenstock)

Table for calculation of focal length and magnification

The following three tables determine the focal length (in [mm]) of a lens, given a subject with breadth G at a distance of g on a 1/3", 1/2" or 2/3" CCD chip. Please note that the format of the selected lens must be larger or at least the same size as the CCD chip.

1/3'' CCD Chip

Working distance g [mm]	Width of subject G [mm]											
	5	10	25	50	75	100	125	150	200	300	400	500
25	12	8										
50	24	16	8									
75	37	24	12									
100	49	32	16	9								
125	61	41	20	11	8							
150	73	49	24	13	9							
175	86	57	28	15	11	8						
200	98	65	32	18	12	9						
300		97	48	26	18	14	11	9				
400			64	35	24	18	15	12	9			
500			81	44	30	23	18	16	12	8		
750				66	45	34	28	23	18	12	9	
1000				88	60	46	37	31	23	16	12	10
1500					90	69	55	47	35	24	18	14
2000						92	74	62	47	31	24	19

1/2'' CCD Chip

Working distance g [mm]	Width of subject G [mm]											
	5	10	25	50	75	100	125	150	200	300	400	500
25	14	10										
50	28	20	10									
75	42	29	15	9								
100	56	39	20	11	8							
125	70	49	25	14	10	8						
150	84	59	31	17	12	9						
175	98	68	36	20	14	11	9					
200		78	41	23	16	12	10	8				
300			61	34	24	18	15	12	9			
400			82	45	31	24	19	16	12	8		
500				57	39	30	24	20	16	10	8	
750				85	59	45	37	31	23	16	12	9
1000					79	60	49	41	31	21	16	13
1500						90	73	61	47	31	24	19
2000							97	82	62	42	31	25

2/3'' CCD Chip

Working distance g [mm]	Width of subject G [mm]											
	5	10	25	50	75	100	125	150	200	300	400	500
25	16	12										
50	32	23	13									
75	48	35	20	11	8							
100	64	47	26	15	11	8						
125	80	59	33	19	13	10	8					
150	96	70	39	22	16	12	10	8				
175		82	46	26	18	14	12	10				
200		94	52	30	21	16	13	11	8			
300			78	45	32	24	20	17	13			
400				60	42	32	26	22	17	11	9	
500				75	53	40	33	28	21	14	11	9
750					79	61	49	42	32	21	16	13
1000						81	66	55	42	28	22	17
1500							99	83	63	43	32	26
2000									84	57	43	35

			Width of subject G [mm]											
			5	10	25	50	75	100	125	150	200	300	400	500
Width of Image	Chip format	1/3" (4.8mm)	0.960	0.480	0.192	0.096	0.064	0.048	0.038	0.032	0.024	0.016	0.012	0.010
		1/2" (6.4mm)	1.280	0.640	0.256	0.128	0.085	0.064	0.051	0.043	0.032	0.021	0.016	0.013
		2/3" (8.8mm)	1.760	0.880	0.352	0.176	0.117	0.008	0.070	0.043	0.044	0.029	0.022	0.018
	Line length	512 (7.2mm)	1.440	0.720	0.288	0.144	0.096	0.072	0.058	0.048	0.036	0.024	0.018	0.014
		1024 (14.3mm)	2.860	1.430	0.572	0.286	0.190	0.143	0.114	0.095	0.072	0.048	0.036	0.029
		2048 (28.7mm)	5.740	2.870	1.148	0.574	0.383	0.287	0.230	0.191	0.144	0.096	0.071	0.057
		4092 (57.3mm)	11.460	5.730	2.292	1.146	0.764	0.573	0.458	0.382	0.287	0.191	0.143	0.115

The choice of some lenses (for example the Rodenstock 35mm lenses) should not be based on their focal length, rather their magnification. The table to the left determines magnification *m=B/G* on various CCD areas and lines. The length of the line sensors is based upon a pixel width of 14µm. The magnification factors with a gray background are larger than 0.1 and therefore lie in the macro range.